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# Tensor Networks for HEP and Quantum Computing

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For more details see [arXiv:2203.04902](https://arxiv.org/abs/2203.04902)

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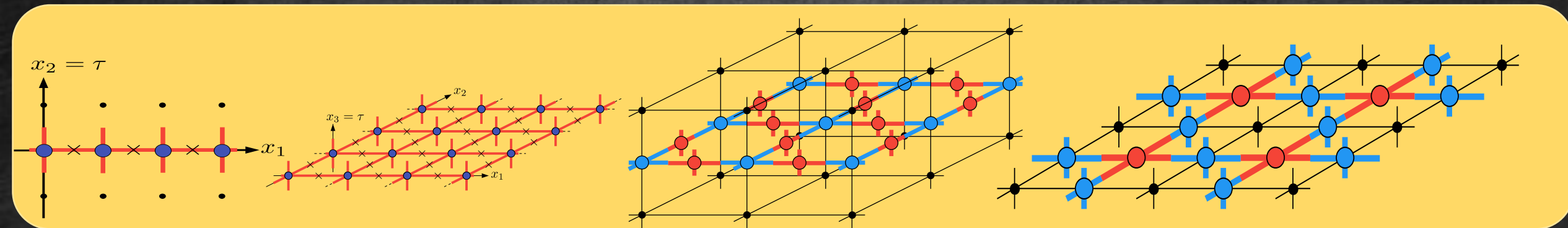
# Tensor Lattice Field Theory (TLFT)

Many tensors in physics: relativistic form factors, space-time curvature, elasticity...

Here, we focus on multi-indices objects that can be used to:

1. Rewrite the partition functions or transfer matrices of lattice gauge theory models

For refs. see: YM, J. Unmuth-Yockey, R. Sakai, Reviews of Modern Physics 94 (2022)



2. Provide compact ways to represent entangled quantum states. Important examples:

Matrix Product States

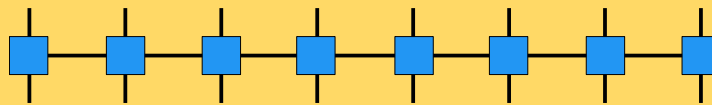
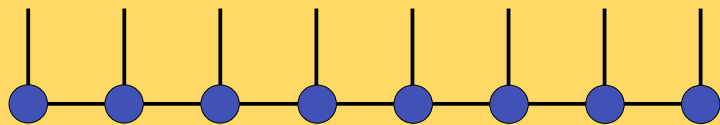
and

Matrix Product Operators

For refs. see

I. Cirac et al.

RMP 93 (2021)



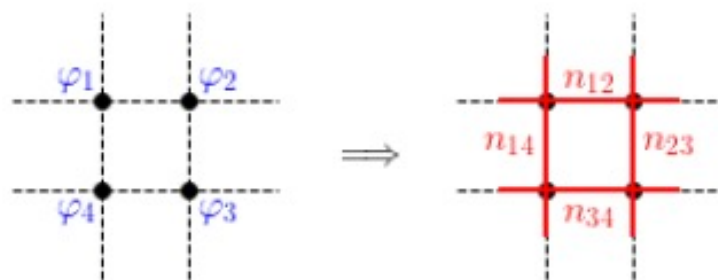
# TrLFT: From compact to discrete (O(2) example)

$$Z_{O(2)} = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} e^{\beta \sum_{x,\mu} \cos(\varphi_{x+\hat{\mu}} - \varphi_x)} = \text{Tr} \prod_x T_{n_{x-\hat{1},1}, n_{x,1}, \dots, n_{x,D}}^{(x)}$$

$$e^{\beta \cos(\varphi_{x+\hat{\mu}} - \varphi_x)} = \sum_{n_{x,\mu}=-\infty}^{\infty} e^{i n_{x,\mu} \varphi_{x+\hat{\mu}}} l_{n_{x,\mu}}(\beta) e^{-i n_{x,\mu} \varphi_x}.$$

Tensor :  $T_{n_{x-\hat{1},1}, n_{x,1}, \dots, n_{x-\hat{D},D}, n_{x,D}}^{(x)} = \sqrt{l_{n_{x-\hat{1},1}} l_{n_{x,1}} \dots l_{n_{x-\hat{D},D}} l_{n_{x,D}}} \times \delta_{n_{x,\text{out}}, n_{x,\text{in}}}$

$$\prod_x \int_{-\pi}^{\pi} d\varphi_x \Rightarrow \sum_{\{n\}}$$



The gauged version is the Abelian Higgs model.



# Compact Abelian Higgs Model (CAHM)

$$Z_{CAHM} = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_{x,\mu}}{2\pi} e^{-S_{gauge} - S_{matter}},$$

$$S_{gauge} = \beta_{pl.} \sum_{x,\mu < \nu} (1 - \cos(A_{x,\mu} + A_{x+\hat{\mu},\nu} - A_{x+\hat{\nu},\mu} - A_{x,\nu})),$$

$$S_{matter} = \beta_l. \sum_{x,\mu} (1 - \cos(\varphi_{x+\hat{\mu}} - \varphi_x + A_{x,\mu})).$$

Gauged version of the O(2) model: the global  $\varphi$  shift becomes local

$$\varphi'_x = \varphi_x + \alpha_x$$

Local changes in  $S_{matter}$  are compensated by

$$A'_{x,\mu} = A_{x,\mu} - (\alpha_{x+\hat{\mu}} - \alpha_x),$$

which leaves  $S_{gauge}$  invariant.

The matter fields can be decoupled by simply setting  $\beta_l. = 0$  (we are left with the pure gauge  $U(1)$  lattice model)



# Fourier expansions and field integrations

Links: 
$$e^{\beta l. \cos(\varphi_{x+\hat{\mu}} - \varphi_x + A_{x,\mu})} = \sum_{n_{x,\mu}=-\infty}^{+\infty} e^{in_{x,\mu}(\varphi_{x+\hat{\mu}} - \varphi_x + A_{x,\mu})} I_{n_{x,\mu}}(\beta l.) ,$$

$\varphi$  integration provides the O(2) selection rule:

$$\sum_{\mu} [-n_{x,\mu} + n_{x-\hat{\mu},\mu}] = 0. \quad (1)$$

Plaquettes: 
$$e^{\beta pl. \cos(A_{x,\mu} + A_{x+\hat{\mu},\nu} - A_{x+\hat{\nu},\mu} - A_{x,\nu})} = \sum_{m_{x,\mu,\nu}=-\infty}^{+\infty} e^{im_{x,\mu,\nu}(A_{x,\mu} + A_{x+\hat{\mu},\nu} - A_{x+\hat{\nu},\mu} - A_{x,\nu})} I_{m_{x,\mu,\nu}}(\beta pl.),$$

$A_{x,\mu}$  integration provides the selection rule:

$$\sum_{\nu > \mu} [m_{x,\mu,\nu} - m_{x-\hat{\nu},\mu,\nu}] + \sum_{\nu < \mu} [-m_{x,\nu,\mu} + m_{x-\hat{\nu},\nu,\mu}] + n_{x,\mu} = 0. \quad (2)$$

Note 1: (2) implies (1) (discrete version of  $\partial_\mu \partial_\nu F^{\mu\nu} = \partial_\mu J^\mu = 0$ .)

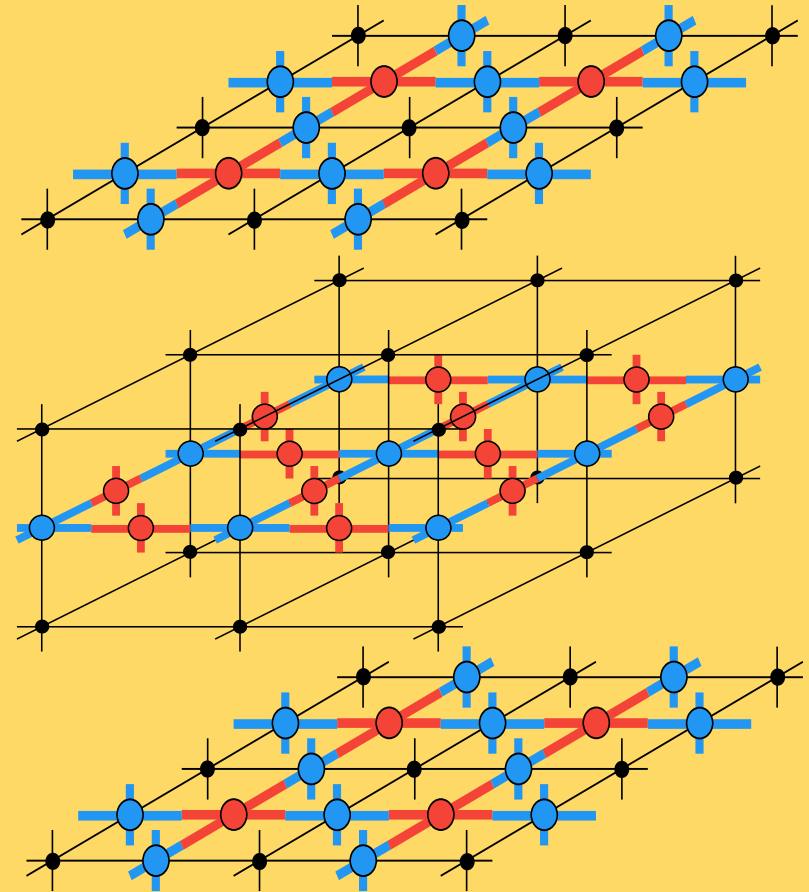
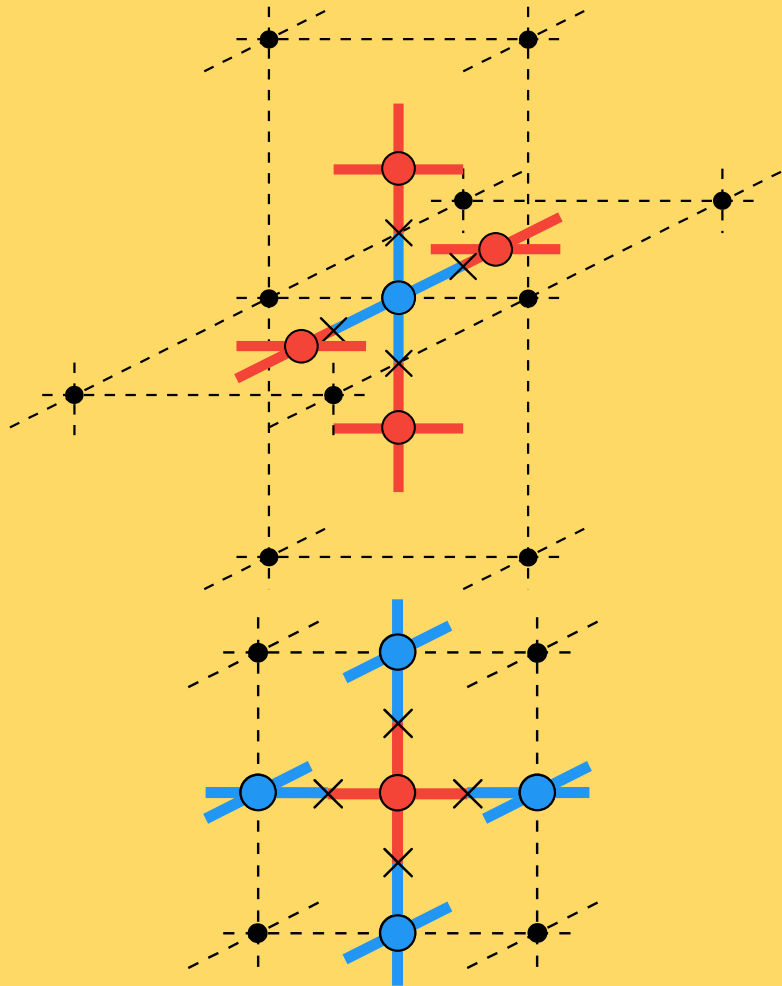
Note 2: Gauge quant. numbers  $m$  determine matter  $n_{x,\mu}(\{m\})$

Note 3: In the unitary gauge,  $\varphi$  disappears



# Tensor assembly in 2+1 D

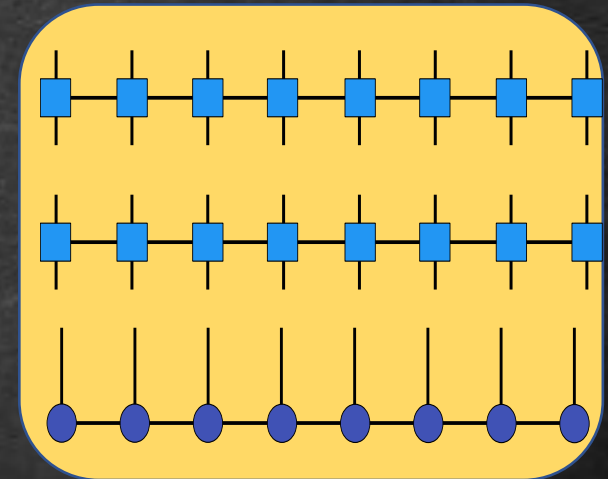
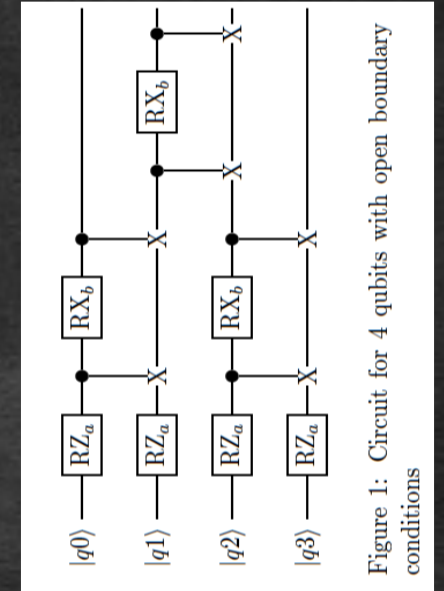
Gauss's law automatically enforced



"Lasagna" transfer matrix

# Tensor methods for quantum computing

- Quantum computing requires complete discretizations
- Known lattice methods are used for **space discretization**
- Discretization of path-integrals by **character expansions**:
  - Are known, "hard integrals" are done (Bessel, Maxwell, ...)
  - Provide **symmetry compatible truncations**
- Tensors are **local** and provide local **effective theories**
- For gauge theories, **gauge invariance** is manifest
- Isolate the **building blocks** of quantum algorithms
- Classical algorithms can be used for **state preparation**
- Tensor network approximations (MPS, PEPS, ..) provide **efficient quantum state tomography**



# Interdisciplinary effort (HEP, NP and CM)

INT Program 21-1c

Tensor Networks in Many Body and Quantum Field Theory

May 17 - June 4, 2021

## Organizers:

- Simon Catterall, Syracuse University, [smcatterall@gmail.com](mailto:smcatterall@gmail.com)
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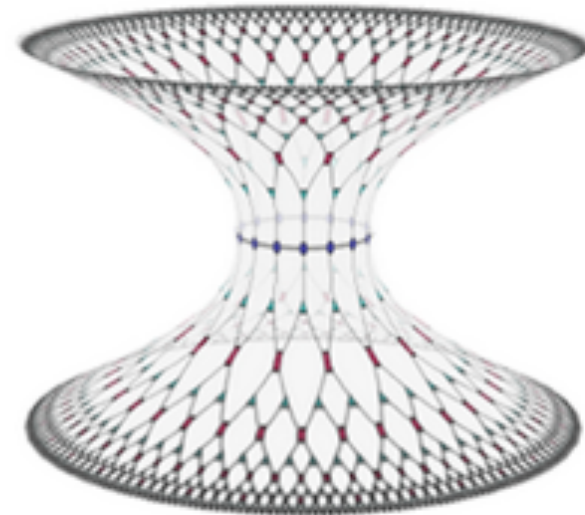
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- Alesha Vertrees, [aleshav@uw.edu](mailto:aleshav@uw.edu), (206) 221-8914

## [Seminar Schedule](#)



# Long term Physics Goals: QCD calculations

Lagrangian methods (numerical coarse graining):

- Validation using existing Monte Carlo results (spectroscopy, form factors)
- Higher accuracy at larger volume (computing time goes like  $\log(V)$ )?
- Finite density calculations for the QCD phase diagram

Hamiltonian methods (smoothly connected to Lagrangian methods)

- Real-time evolution
- Connection with digital and analog quantum computing
- Jet physics: hybrid QuPythia algorithms?
- Out of equilibrium processes

Time scale: 5–10 years with enough practitioners?

# Our roadmap: the “Kogut ladder”

## An introduction to lattice gauge theory and spin systems\*

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This article is an interdisciplinary review of lattice gauge theory and spin systems. It discusses the fundamentals, both physics and formalism, of these related subjects. Spin systems are models of magnetism and phase transitions. Lattice gauge theories are cutoff formulations of gauge theories of strongly interacting particles. Statistical mechanics and field theory are closely related subjects, and the connections between them are developed here by using the transfer matrix. Phase diagrams and critical points of continuous transitions are stressed as the keys to understanding the character and continuum limits of lattice theories. Concepts such as duality, kink condensation, and the existence of a local, relativistic field theory at a critical point of a lattice theory are illustrated in a thorough discussion of the two-dimensional Ising model. Theories with exact local (gauge) symmetries are introduced following Wegner's Ising lattice gauge theory. Its gauge-invariant “loop” correlation function is discussed in detail. Three-dimensional Ising gauge theory is studied thoroughly. The renormalization group of the two-dimensional planar model is presented as an illustration of a phase transition driven by the condensation of topological excitations. Parallels are drawn to Abelian lattice gauge theory in four dimensions. Non-Abelian gauge theories are introduced and the possibility of quark confinement is discussed. Asymptotic freedom of  $O(n)$  Heisenberg spin systems in two dimensions is verified for  $n \geq 4$  and is explained in simple terms. The direction of present-day research is briefly reviewed.

### CONTENTS

I. Introduction—An Overview of this Article	659	C. The planar model in the periodic Gaussian approximation	698
II. Phenomenology and Physics of Phase Transitions	661	D. Renormalization group analysis and the theory's critical region	700
A. Facts about critical behavior	661	VIII. Non-Abelian Lattice Theories	705
B. Correlation length scaling and the droplet picture	663	A. General formulation of the $SU(2)$ theory	705
III. The Transfer Matrix—Field Theory and Statistical Mechanics	664	B. Special features of the non-Abelian theory	706
A. General remarks	664	C. Renormalization group analysis of $O(n)$ spin systems in two dimensions	708
B. The path integral and transfer matrix of the simple harmonic oscillator	664	D. Results from the Migdal recursion relation	710
C. The transfer matrix for field theories	666	IX. Parting Comments	710
IV. The Two-Dimensional Ising Model	669	Acknowledgment	713
A. Transfer matrix and $\tau$ -continuum formulation	669	References	713
B. Self-duality of the Ising model	671		
C. Strong coupling expansions for the mass gap, weak coupling expansions for the magnetization	672		
D. Kink condensation and disorder	676		
E. Self-duality of the isotropic Ising model	677		
F. Exact solution of the Ising model in two dimensions	678		
V. Wegner's Ising Lattice Gauge Theory	681		
A. Global symmetries, local symmetries, and the energetics of spontaneous symmetry breaking	681		
B. Constructing an Ising model with a local symmetry	682		

### I. INTRODUCTION—AN OVERVIEW OF THIS ARTICLE

This article consists of a series of introductory lectures on lattice gauge theory and spin systems. It is intended to explain some of the essentials of these subjects to students interested in the field and research physicists whose expertise lies in other domains. The expert in lattice gauge theory will find little new in the following pages aside from the author's personal perspective and overview. The style of this presentation

REVIEWS OF MODERN PHYSICS, VOLUME 94, APRIL–JUNE 2022

## Tensor lattice field theory for renormalization and quantum computing

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(published 26 May 2022)

The successes and limitations of statistical sampling for a sequence of models studied in the context of lattice QCD are discussed and the need for new methods to deal with finite-density and real-time evolution is emphasized. It is shown that these lattice models can be reformulated using tensorial methods where the field integrations in the path-integral formalism are replaced by discrete sums. These formulations involve various types of duality and provide exact coarse-graining formulas that can be combined with truncations to obtain practical implementations of the Wilson renormalization group program. Tensor reformulations are naturally discrete and provide manageable transfer matrices. Truncations with the time continuum limit are combined, and Hamiltonians suitable for performing quantum simulation experiments, for instance, using cold atoms, or to be programmed on existing quantum computers, are derived. Recent progress concerning the tensor field theory treatment of noncompact scalar models, supersymmetric models, economical four-dimensional algorithms, noise-robust enforcement of Gauss's law, symmetry preserving truncations, and topological considerations are reviewed. Connections with other tensor network approaches are also discussed.

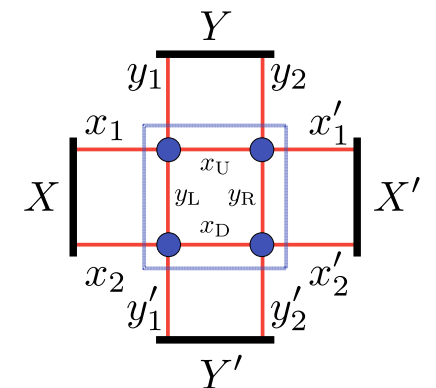
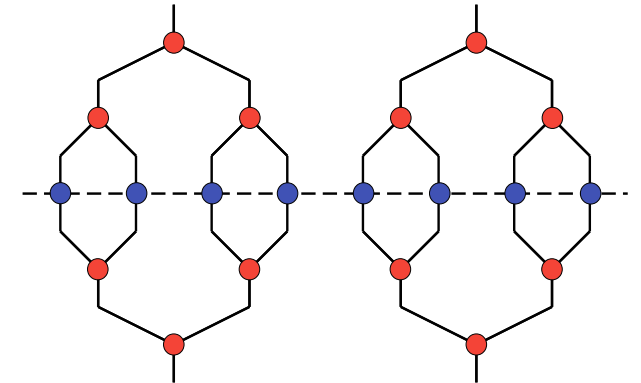
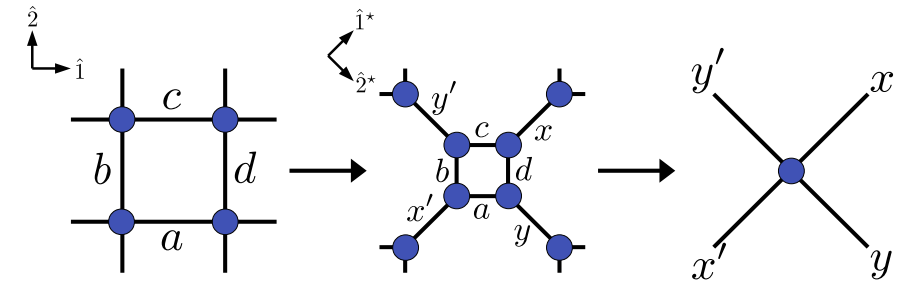
DOI: 10.1103/RevModPhys.94.025005

### CONTENTS

I. Introduction	2	D. Exact blocking	19
II. Lattice Field Theory	4	VI. Tensor Renormalization Group	20
A. The Kogut sequence: From Ising to QCD	4	A. Block spinning through SVD	20
B. Classical lattice models and path integral	4	B. Optimized truncations	20
C. Physical applications	7	C. Higher-dimensional algorithms	22
D. Computational methods beyond perturbation theory	7	D. Observables with tensors	22
III. Quantum Computing	8	E. Niemeijer–van Leeuwen equation	23
A. Situations where importance sampling fails	8	F. A simple example of TRG fixed point	24
		G. Corner double line structure on tensor network	25

# Coarse Graining: TRG, HOTRG, ATRG, ...

- Do not rely on statistical sampling (classic, deterministic)
- Deal well with sign problems
- Resource intensive (polynomials with large powers)
- Most lattice field theory models can be reformulated using Tensor Lattice Field Theory
- Current numerical implementations in 1+1 and 2+1 dimensions involve: spin and gauge models with Abelian and non-Abelian symmetries, scalar theories, staggered and Wilson fermions, supersymmetric models
- Not currently feasible for complex theories in 3+1 dimensions
- For refs. see: YM, J. Unmuth-Yockey, R. Sakai, *Reviews of Modern Physics* 94 (2022)

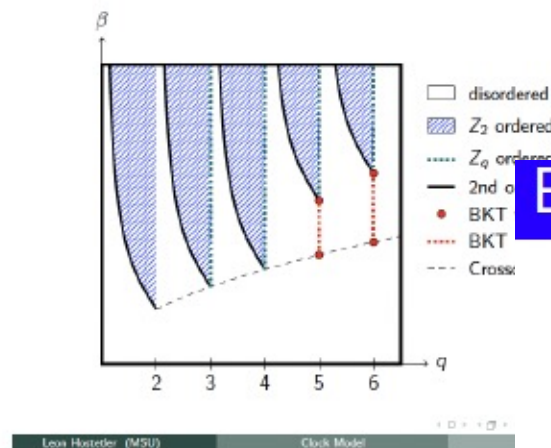


# Applications to studies of quantum simulators (QuEra)

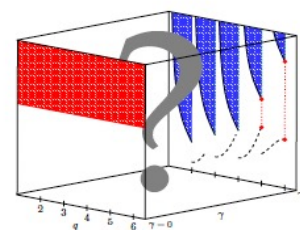
## Interpolation among $Z_q$ clock models

with Leon Hostetler, Jin Zhang, Ryo Sakai, Judah Unmuth-Yockey, and Alexei Bazavov PRD 104 054505

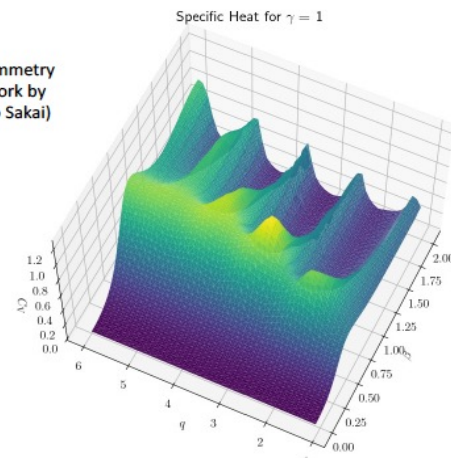
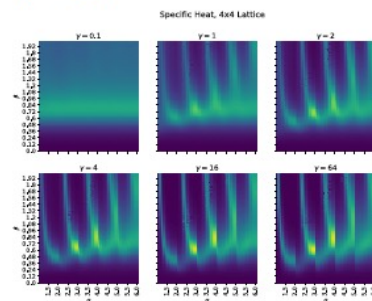
- $O(2)$  model with Symm. breaking :  $\Delta S = \gamma \sum_x \cos(q\varphi_x)$
- $\gamma \rightarrow \infty$ :  $\varphi = \frac{2\pi k}{q}$   $k = 0, 1, \dots, [q]$
- Integer  $q$ :  $Z_q$  symmetry
- Non-integer  $q$ :  $Z_2$  symmetry
- Phase diagram: see right panel



Leon Hostetler (MSU) Clock Model

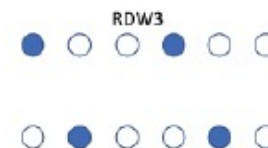
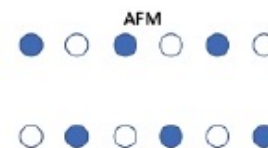
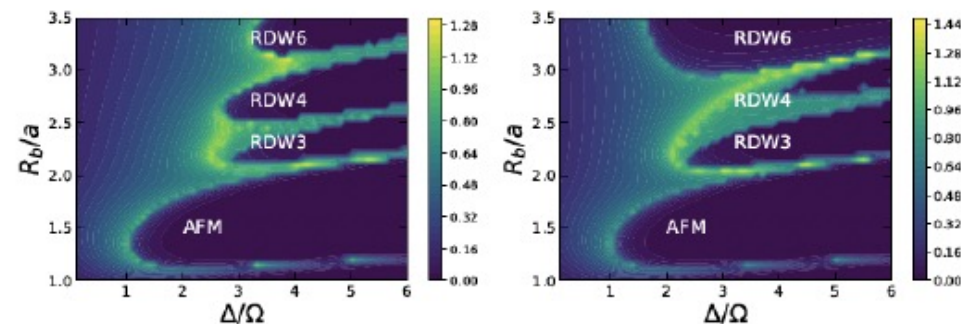


Phase diagram with symmetry breaking (numerical work by Leon Hostetler and Ryo Sakai)



## Example of phase diagram (Jin Zhang)

### 4.2 Two-leg ladder: $dx = dy/2$



## Implementation with Rydberg arrays?

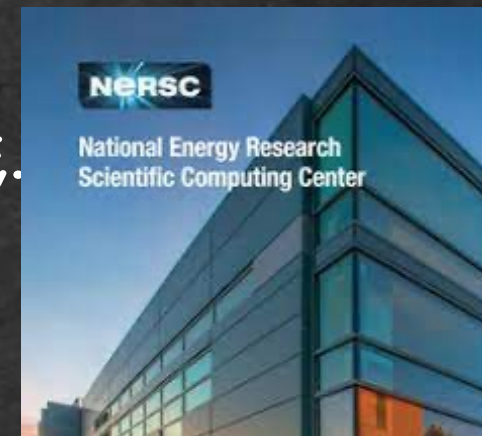
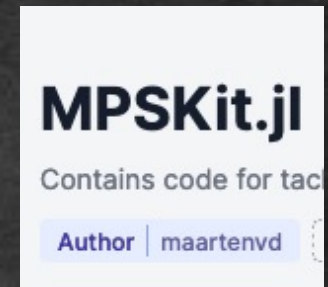
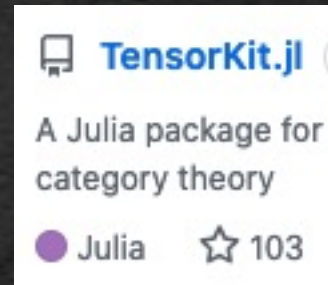
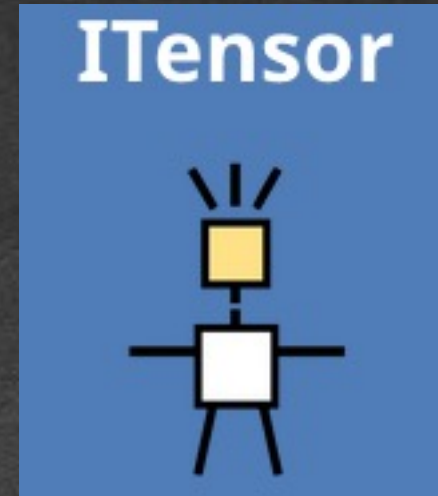
A. Keesling, .., M. Lukin et al. Nature 568: 1D array of  $^{87}\text{Rb}$  atoms evenly separated by a controllable distance, homogeneously coupled to the excited Rydberg state  $|r\rangle$  with detuning  $\Delta$ .

$$H = \frac{\Omega}{2} \sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta \sum_i n_i + \sum_{i < j} V_{ij} n_i n_j$$

For  $R_b/a \simeq q$  integer and  $\Delta$  large enough:  $Z_q$  ordering.

# Software and resources needed

- Tensor libraries with:
  - Reshaping the tensor and permuting the order of indices
  - Element-wise operations
  - Contracting indices
  - Tensor decompositions (SVD or eigen-decomposition)
- Distributed memory
- Implementations of symmetries
- Access to large scale classical computers: NERSC, USQCD, ALCF,
- Development of software ecosystem and workforce



# Credits and references

We apologize for the lack of detailed citations in this short talk. The results presented here are due to discussions with or reading the work of:

M. C. Banuls, Tao Xiang, X.-G. Wen, L. P. Yang, S. Shandrasekharan, S.-W. Tsai, Jin Zhang, U. Schollwöck, L. Kadanoff, N. Schuch, M. Levine, C. Gatttringer, J. Bloch, M. Hite, Z. Y. Xie, H. Zou, Y. Liu, F. Verstraete, G. Evenbly, A. Roggero, J. Haegeman, D. Kadoh, Y. Kuramashi, Y. Nakamura, S. Takeda, Y. Yoshimura, D. Adachi, T. Okubo, S. Todo, N. Butt, R. G. Jha, M. Asaduzzaman, Y. Shimizu, A. Celi, I. Cirac, M. Dalmonte, L. Fallani, K. Jansen, G. Vidal, M. Lewenstein, S. Montangero, C.A. Muschik, B. Reznik, E. Rico, L. Tagliacozzo, K. Van Acoleyen, U.-J. Wiese, R. Brower, D. Berenstein, A. Bazavov, M. Wingate, E. Zohar, and P. Zoller. We apologize for omissions especially regarding early work (e.g., M. Fannes et al., S. White, T. Nishino et al. ...)

For detailed lists of references, see e.g.,

M.C. Banuls et al. Eur. Phys. J. D 74 (2020)

I. Cirac et al. Rev. Mod. Phys. 93 (2021)

YM et al. Rev. Mod. Phys. 94 (2022)

# Lattice Gauge Theory at Universities?

- Recent retirements of lattice gauge theorists working at Universities
- Physics Departments are typically not keen to hire in lattice gauge theory
- Senior physicists involved in departmental reviews do not always appreciate the crucial contributions of lattice gauge theory to the HEP community
- Problematic to maintain continuity in some important areas (e. g. flavor physics)
- Physics Departments are typically more likely to hire in quantum information/computation areas
- Examples of junior faculty successfully engaged in traditional lattice gauge theory **and** quantum computation
- **Funding for Tensor Lattice Field Theory research and related areas could be an incentive for departments to hire lattice gauge theorists**

# Conclusions

- Tensor networks for HEP and quantum computing : **big goals (QCD) with manageable steps** (the “Kogut ladder”)
- **Interdisciplinary effort**, with large input from condensed matter
- **Tensor Lattice Field Theory** is a generic tool to discretize path integrals:
  - applies to most most lattice models
  - gauge-invariant approach
  - truncations respect symmetries (but their critical effects are subtle)
  - smooth connection between Lagrangian and Hamiltonian approaches
  - rapidly evolving research area
- **Coarse graining**: friendly competitor/validator to quantum computing
- Access to **large computing facilities** is important
- Needs to develop **tensor libraries, software and a larger community in US**
- **Funding for Tensor Lattice Field Theory** would help maintaining a lattice gauge theory presence at **Universities**
- Thanks for listening!